An Estimator for statistical anisotropy from the CMB bispectrum

(based on N.Bartolo, E.D, M.Liguori S.Matarrese and A.Riotto, to appear)

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Statistical anisotropy

- Observational hints of statistical anisotropy from Cosmic Microwave Background (CMB) data analysis
  (e.g. cold spots; low amplitude of the quadrupole moment; alignment between quadrupole and octupole; lack of large angular scale CMB power; dipolar and quadrupolar power asymmetry.)
- Primordial vector field models of inflation predicting statistical anisotropy in the power spectrum and in higher order cosmological correlators
  (e.g. hybrid inflation, vector curvaton, $f(\phi)$ models.)

$$P(\vec{k}) = P^{iso}(k)\left[1 + g(k)\left(\hat{k}\cdot\hat{n}\right)^2\right] \quad \rightarrow \quad g = 0.29 \pm 0.031, \;\; \| \; ecliptic\ poles$$

(astro-ph/0701357,arXiv:0709.1144) $(arXiv: 0807.2242, 0908.0963, 0911.0150)$

Complementary bispectrum analysis:

$$B(\vec{k}_1, \vec{k}_2, \vec{k}_3) = B^{iso}(k_1, k_2)\Big[1 + \Gamma\left(\hat{k}_1\cdot\hat{N}\right)^2 + \Delta\left(\hat{k}_2\cdot\hat{N}\right)^2 + \Theta\left(\hat{k}_1\cdot\hat{N}\right)^2\left(\hat{k}_2\cdot\hat{N}\right)^2 + \Omega\left(\hat{k}_1\cdot\hat{k}_2\right)\left(\hat{k}_1\cdot\hat{N}\right)\left(\hat{k}_2\cdot\hat{N}\right)\Big] + 2\ perms.$$

It turns out that: there are some models where the anisotropy of the power spectrum is not observable, whereas the anisotropic amplitude of the bispectrum can be relatively large!

Anisotropic CMB bispectrum

$$B^{l_1\ l_2\ l_3}_{m_1 m_2 m_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = (4\pi)^3 (-i)^{l_1+l_2+l_3} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} \Delta_{l_1}(k_1)\Delta_{l_2}(k_2)\Delta_{l_3}(k_3)$$
$$\times Y_{l_1 m_1}(\hat{k}_1) Y_{l_2 m_2}(\hat{k}_2) Y_{l_3 m_3}(\hat{k}_3) \langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle$$

- Spherical harmonics expansion of temperature anisotropies:
  $\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$
  $a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \Delta_l(k)\Phi(\vec{k}) Y_{lm}(\hat{k})$
- Primordial bispectrum:
  $\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3) B(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \left(\frac{5}{3}\right)^3 \langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle$
- Simplest template:
  $B(\vec{k}_1,\vec{k}_2,\vec{k}_3) = B^{iso}(k_1,k_2)\left[1 + g_1 \left(\hat{k}_1\cdot\hat{n}\right)^2 + g_2 \left(\hat{k}_2\cdot\hat{n}\right)^2\right] + \ perms.$
  $\rightarrow g_1 \left(\hat{k}_1\cdot\hat{n}\right)^2 = \sum_{LM} \lambda_{LM} Y_{LM}(\hat{k}_1)$

⇓

$$B^{l_1\ l_2\ l_3}_{m_1 m_2 m_3} = B^{l_1 l_2 l_3 (I)}_{m_1 m_2 m_3} + B^{l_1 l_2 l_3 (A)}_{m_1 m_2 m_3} = f_{NL}\left(B^{l_1 l_2 l_3 (I)}_{m_1 m_2 m_3}\Big|_{f_{NL}=1} + \sum_{L\geq 2, M} \lambda_{LM} B^{l_1 l_2 l_3 (A) LM}_{m_1 m_2 m_3}\Big|_{f_{NL}=1}\right)$$

from isotropic and anisotropic contributions:

$$B^{l_1 l_2 l_3 (I)}_{m_1 m_2 m_3} = b_{l_1 l_2 l_3} G^{l_1 l_2 l_3}_{m_1 m_2 m_3} \equiv \left(\frac{3}{5}\right)^3 \left(\frac{2}{\pi}\right)^3 \int dx x^2 \int dk_1 dk_2 dk_3 (k_1 k_2 k_3)^2 \Delta_{l_1}(k_1)\Delta_{l_2}(k_2)\Delta_{l_3}(k_3)$$
$$\times j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x) B^{iso}(k_1,k_2,k_3) G^{l_1 l_2 l_3}_{m_1 m_2 m_3}$$

$$B^{l_1 l_2 l_3 (A) LM}_{m_1 m_2 m_3} = \left(\frac{3\times 6}{5\pi}\right)^3 \sum_{l_1' m_1'} G^{l_1' l_2 l_3}_{m_1' m_2 m_3} G^{l_1 l_1' L}_{m_1 -m_1' M} (-1)^{l_1} (i)^{l_1+l_1'} (-1)^{m_1'} b_{l_1' l_2 l_3} + 2 \ perms.$$

Results and conclusions

Estimators and Fisher matrix for our anisotropy parameters:

$$\hat{\lambda}_{LM} = \frac{1}{F_{\lambda_{LM}\lambda_{LM}}} \left(f_{NL} B^{(A)LM} - f_{NL}^2 B^{(I)} B^{(A)LM} \right)$$

$$F_{\lambda_{LM}\lambda_{LM}} = f_{NL}^2 \left\langle B^{(A)LM} B^{(A)L-M} (-1)^M \right\rangle$$

$$\mathrm{B}^{(I)} \equiv \frac{1}{6} \sum_{l_i m_j} B^{l_1 l_2 l_3 (I)}_{m_1 m_2 m_3}|_{f_{NL}=1} \left(\frac{a^*_{l_1 m_1} a^*_{l_2 m_2} a^*_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}} - \frac{(-1)^{m_2}}{C_{l_1} C_{l_2}} \delta_{l_2 l_3} \delta_{m_2 - m_3} a^*_{l_1 m_1} - 2 \; perms. \right)$$

$$B^{(A)LM} \equiv \frac{1}{6} \sum_{l_i m_j} B^{l_1 l_2 l_3 (A)LM}_{m_1 m_2 m_3}|_{f_{NL}=1} \left(\frac{a^*_{l_1 m_1} a^*_{l_2 m_2} a^*_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}} - \frac{(-1)^{m_2}}{C_{l_1} C_{l_2}} \delta_{l_2 l_3} \delta_{m_2 - m_3} a^*_{l_1 m_1} - 2 \; perms. \right)$$

1σ error for the quadrupole statistical anisotropy in the bispectrum (preliminary results):

- $\frac{1}{\sigma_{\lambda_{2M}}} \simeq 0.1 \frac{l^5 b_{lll}}{\left(l^2 C_l\right)^{3/2}} \simeq 0.77 f_{NL} \left(\frac{l}{2000}\right)$
- $\sigma_{\lambda_{2M}} \simeq 0.04$ for $f_{NL} = 32$ and $l_{max} \simeq 2000$.

→ Primordial vector field models exist that predict a bispectrum statistical anisotropy that might be large enough to be within the sensitivity ranges of forthcoming experiments. However, a careful analysis of systematic errors and of other possible cosmological signals that can mimic statistical anisotropy will also be needed!